Geometric Folding Algorithms: Bridging Theory to Practice

PI: Prof. Erik D. Demaine, MIT

Final report

1 Results

Research followed two main thrusts:

1.1 Reconfigurable Robots

- Solved the hinged dissection problem, which was over a 100 years old, proving that any finite collection of shapes have a hinged dissection.
- Proved that crystalline robots can reconfigure extremely efficiently: O(log n) time and O(n) moves.
- Proved that any orthogonal polyhedron can be folded from a single, universal crease pattern (box pleating).

1.2 Origami Design

- Developed mathematical theory for what happens in paper between creases, in particular for the case of circular creases.
- Circular crease origami on permanent exhibition at MoMA in New York.
- Developing mathematical theory of Tomohiro Tachi's Origamizer framework for efficiently folding any polyhedron from a sheet of paper.
- Developing mathematical theory of Robert Lang's TreeMaker framework for efficiently folding tree-shaped origami "bases".
- Developing tools with Tomohiro Tachi for animation of real origami.
- Visited Japan in December 2007, sharing latest results in computational origami.

2 Publications

2.1 Book

"Geometric Folding Algorithms: Linkages, Origami, Polyhedra" (joint work with Joseph O'Rourke), Cambridge University Press, July 2007.

2.2 Journal Papers

"Refolding Planar Polygons" (joint work with Hayley N. Iben and James F. O'Brien), *Discrete & Computational Geometry*, to appear. Special issue of selected papers from SoCG 2006.

"Grid Vertex-Unfolding Orthostacks" (joint work with John Iacono and Stefan Langerman), *International Journal of Computational Geometry and Applications*, to appear.

"Staged Self-Assembly: Nanomanufacture of Arbitrary Shapes with $O(1)$ Glues" (joint work with Martin L. Demaine, Sandor P. Fekete, Mashhood Ishaque, Eynat Rafalin, Robert T. Schweller, and Diane L. Souvaine), *Natural Computing*, volume 7, number 3, pages 347–370, Sept. 2008. Special issue of selected papers from DNA 2007.

"Edge-Unfolding Nested Polyhedral Bands" (joint work with Greg Aloupis, Stefan Langerman, Pat Morin, Joseph O'Rourke, Ileana Streinu, and Godfried Toussaint), *Computational Geometry: Theory and Applications*, volume 39, number 1, pages 30–42, Jan. 2008.

2.3 Book Chapters

"All Polygons Flip Finitely... Right?" (joint work with Blaise Gassend, Joseph O'Rourke, and Godfried T. Toussaint), in *Surveys on Discrete and Computational Geometry: Twenty Years Later*, J. Goodman, J. Pach, and R. Pollack, eds., Contemporary Mathematics 453, pages 231–255, 2008, American Mathematical Society.

2.4 Conference Papers

"Reconfiguration of Cube-Style Modular Robots Using O(log n) Parallel Moves" (joint work with Greg Aloupis, Sebastien Collette, Stefan Langerman, Vera Sacristan, and Stefanie Wuhrer), in *Proceedings of the 19th Annual International Symposium on Algorithms and Computation*, to appear, Dec. 2008.

"Hinged Dissections Exist" (joint work with Timothy G. Abbott, Zachary Abel, David Charlton, Martin L. Demaine, and Scott D. Kominers), in *Proceedings of the 24th Annual ACM Symposium on Computational Geometry*, pages 110–119, College Park, MD, June 2008.

"Linear Reconfiguration of Cube-Style Modular Robots" (joint work with Greg Aloupis, Sebastien Collette, Mirela Damian, Robin Flatland, Stefan Langerman, Joseph O'Rourke, Suneeta Ramaswami, Vera Sacristan, and Stefanie Wuhrer), in *Proceedings of the 18th Annual International Symposium on Algorithms and Computation*, pages 208–219, Dec. 2007.

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I. RECONFIGURABLE ROBOTS
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* Proved that any orthogonal polyhedron can be folded from a single, universal crease pattern (box pleating).
II. ORIGAMI DESIGN
* Developed mathematical theory for what happens in paper between creases, in particular for the case of circular creases.
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